Network-Constrained Adaptive Control with a Nonlinear Tracking Function

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***Abstract*— We consider a problem where several agents need to achieve a common control objective. The motivation for this problem is control of power generation in a wind farm under weak-grid conditions, with the individual turbine-generators acting as agents. The control objective is a nonlinear function of the state of the agents. Furthermore, this nonlinear function depends on the parameters of the network connecting the agents, and these parameters are unknown. Based on a newly developed nonlinear adaptive control methodology, a process is designed that simultaneously learns the network parameters and, using individual controllers, drives the agents to their respective steady-state solutions. We also compare the performance of this design to an empirical control design that requires less communication.**

I. Introduction

A typical problem in controlling a wind farm, a network of several turbines, is to regulate the power produced by each turbine. The active power is regulated so as to extract the maximum power available from the wind, and the reactive power is regulated such that losses are minimized and stability is ensured. There have been numerous publications, such as [1], presenting similar control approaches for addressing this problem at the individual turbine level. Using individual controllers, without any communication between the turbines, is sufficient under certain strong-grid conditions (see [2] for the definition). When these conditions are not met, a coordinated network-control approach with judicious communication between the individual turbines is called for, and results in a significantly more efficient operation. The latter is the focus of this paper.

Each agent in the network, corresponding to an individual wind turbine-generator, is a dynamical system with a controllable state — the electric current. The signal to be regulated is power, a nonlinear function of the accumulated state of all agents and of the network parameters. We further assume that the network parameters are unknown.

To solve this problem we propose to use a newly developed nonlinear adaptive control methodology [3]. It was originally developed for non-networked systems, and is extended in this paper to networked systems. The approach used by this adaptive controller consists of simultaneously estimating the network parameters, anticipating the desired steady-state solution, and using individual controllers to drive the agents to the desired state. As is typical in adaptive control, we prove convergence to the steady-state solution without requiring that the estimation error goes to zero.

The main contribution of this paper is the theoretical validation of the new methodology in a network control framework, a methodology with clear advantages over other solutions. As such, a simplified wind turbine model is used in order to make the point. It should be noted that this methodology is also being successfully validated using a comprehensive wind turbine model (a paper reporting these results is in preparation).

A. Notations

We use j for $\sqrt{-1}$, $|\cdot|$ for the absolute value of a real or complex number, $\bar{\cdot}$ for the conjugate operation of a complex number. and $\mathrm{Re}\,\cdot$ and $\mathrm{Im}\,\cdot$ for the real and imaginary parts of a complex number, respectively. We use $\|\cdot\|$ for the 2-norm of a real vector, and $\|\cdot\|_F$ for the Frobenius norm of a matrix. For $n \in \mathbb{N}$ we define $[n] \doteq \{1,\ldots,n\}$. For a vector γ, the matrix $\mathrm{diag}\,(\gamma)$ is a diagonal matrix whose diagonal elements are the elements of γ.

II. Problem Statement

We use complex numbers to simplify the presentation. Let n be the number of agents. The state of the system is given by $x(t)$, $x : \mathbb{R} \to \mathbb{C}^{n+1}$. The state of each agent i, $i \in [n]$, is x_i. The state element x_{n+1} is associated with a pseudo-agent. Each agent $i \in [n]$ is equipped with an individual inner-loop controller which accepts an external reference signal, $u_i \in \mathbb{C}$, such that the resulting closed-loop dynamics of x_i,

$$\dot{x}_i = f_i\left(x_i, u_i\right), \tag{1}$$

satisfies

$$\lim_{t\to\infty} x_i(t) = \lim_{t\to\infty} u_i(t) \tag{2}$$

whenever the second limit exists. The state of the pseudo-agent, x_{n+1}, is varying but not controllable. The dynamical model in (1)-(2) is a general model applicable to many systems, including wind turbine-generators. For the purpose of demonstration, as we are interested in the interaction between the agents rather than the particular dynamics within each agent, we simplify (1) as

$$a\dot{x}_i = -x_i + u_i, \tag{3}$$

where $a \in \mathbb{R}_{>0}$.

In addition to x, a dependent variable $y(t)$, $y : \mathbb{R} \to \mathbb{C}^{n+1}$, exists which is linearly dependent on x and $\dot{x}$:

$$y(t) = Hx(t) + L\dot{x}(t). \tag{4}$$

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The network parameters are encapsulated in H and L, which are unknown. Each agent including the pseudo-agent $i \in [n+1]$ can measure its own x_i and y_i.

The control objective is to have

$$\lim_{t\to\infty} c\left(x(t), y(t)\right) = \lim_{t\to\infty} r(t) \tag{5}$$

where $c : \mathbb{C}^{n+1} \times \mathbb{C}^{n+1} \to \mathbb{R}^{2n+2}$ is defined as

$$c\left(x, y\right) = \left[cp\left(x, y\right)^T, cq\left(x, y\right)^T, \operatorname{Re} x_{n+1}, \operatorname{Im} x_{n+1}\right]^T,$$

$$cp_i\left(x, y\right) = \operatorname{Re} y_i \bar{x}_i, \qquad \forall i \in [n], \tag{6}$$

$$cq_1\left(x, y\right) = \operatorname{Im} x_{n+1} \bar{y}_{n+1}, \tag{7}$$

$$cq_i\left(x, y\right) = \frac{\operatorname{Im} y_{i-1}\bar{x}_{i-1}}{\operatorname{Re} y_{i-1}\bar{x}_{i-1}} - \frac{\operatorname{Im} y_i \bar{x}_i}{\operatorname{Re} y_i \bar{x}_i}, \qquad \forall i \in \{2, \ldots, n\}, \tag{8}$$

and $r(t) \in \mathcal{R}(t) \subset \mathbb{R}^{2n+2}$ is defined as

$$\mathcal{R}(t) \doteq \left\{ r \in \mathbb{R}^{2n+2} \;\middle|\; \begin{array}{l} r_i = p_i \geq 0, \quad \forall i \in [n] \\ r_i = 0, \quad \forall i \in \{n+1, \ldots, 2n\} \\ r_{2n+1} = \operatorname{Re} x_{n+1}, \\ r_{2n+2} = \operatorname{Im} x_{n+1} \end{array} \right\} \tag{9}$$

for some $p_i \geq 0$, $i \in [n]$.

The relation to wind farm control is as follows. Each agent represents a wind turbine-generator. For each $i \in [n]$, the state x_i is the current injected into the grid by the i'th generator, and y_i is the voltage applied on this generator. Therefore $\operatorname{Re} y_i \bar{x}_i$ is the active power generated by generator $i \in [n]$, and $\operatorname{Im} y_i \bar{x}_i$ is the reactive power. Type 3 (equipped with doubly-fed induction generators) and Type 4 turbines use AC-DC-AC converters which allow control of the current injected into the grid [4]. We use the simplified dynamics (3) to approximate the more complicated real dynamics of wind turbines. The pseudo-agent represents the point of connection to the main grid. The state x_{n+1} is the voltage at this point, and y_{n+1} is the current injected into the main grid at this point. Note the reverse in the type of physical quantities that x_{n+1} and y_{n+1} represent compared to those x_i and y_i represent for $i \in [n]$.

Our choice of the function c and the reference signal r is motivated as follows. By driving the active power in (6) to p_i and setting the reference active power p_i to be a certain function of the rotation speed of the turbine i, we can maximize the power extracted from the available wind [1]. The combination of (8) and (9) guarantees that the burden of generating reactive power is distributed evenly among all the generators as a ratio of their active power generation. Driving cq_1 in (7) to zero ensures that no reactive power arrives at the point of connection to the main grid. This is done to maximize the amount of active power delivered to the grid. It is a simple exercise to show that with a single turbine, having no reactive power arriving at the main grid results in maximum active power being delivered, regardless of the line impedance. This is no longer true with several turbines, under the constraint that the burden of reactive power generation be shared equally. In this case the optimal amount of reactive power to arrive at the main grid becomes a function of the unknown H. However, empirical evidence shows that keeping the reactive power at zero provides almost optimal results. We note that while we can compute an optimal reactive power based on an estimation of H, the theory in [3] requires an objective function that is independent of the unknown parameters. This choice also allows the comparison with the approach presented in §IV.

It is important to note that the amount of reactive power that each generator must generate depends on the total active power produced by all the generators, as well as on the unknown network parameters. Therefore at least some communication is necessary for the individual generators to evaluate the required reactive power.

We now focus on the special structure of the unknown parameter H. Each pair of current x_i and voltage y_i is measured in a d-q rotating reference frame, and the reference frames of the different generators are not synchronized. For each $i \in [n]$ let $\gamma_i \in \mathbb{C}$, $|\gamma_i| = 1$, be the transformation between agent's i reference frame and the reference frame in which x_{n+1} and y_{n+1} are measured. Thus $x_i' = \gamma_i x_i$, $y_i' = \gamma_i y_i$, $\forall i \in [n]$, are the corresponding currents and voltages in a synchronized reference frame. Each voltage, y_i', $i \in [n]$, is a linear function of the currents injected by all the generators and the voltage at the point of connection to the main grid:

$$y_i' = \sum_{j\in[n]} z_{ij} x_j' + x_{n+1}, \qquad \forall i \in [n].$$

Assuming no shunt devices and neglecting capacitive leakage of the transmission lines, the current y_{n+1} is the sum of all the other currents. Therefore H from (4) has the special structure

$$H = \begin{bmatrix} z_{11}\gamma_1\bar{\gamma}_1 & \cdots & z_{1n}\gamma_n\bar{\gamma}_1 & \bar{\gamma}_1 \\ \vdots & \ddots & \vdots & \vdots \\ z_{n1}\gamma_1\bar{\gamma}_n & \cdots & z_{nn}\gamma_n\bar{\gamma}_n & \bar{\gamma}_n \\ \gamma_1 & \cdots & \gamma_n & 0 \end{bmatrix}. \tag{10}$$

We note that in the above $1/\gamma_i = \bar{\gamma}_i$, as $|\gamma_i| = 1$.

In the next section we show how to solve this nonlinear control problem.

III. Nonlinear Adaptive Control Approach

In [3] we considered a general, single-agent control problem described by (1), $y = Hx$, and (5), with arbitrary $f(\cdot, \cdot)$ and $c(\cdot, \cdot)$. For this type of problem we developed in [3] a nonlinear adaptive control methodology consisting of three components: an *adaptive law*, a *high-level control law*, and a *low-lever controller*. Both the *adaptive law* and the *high-level control law* work in discrete time using a sampled version of x, y and r: $x[k] \doteq x(kT_s)$, $y[k] \doteq y(kT_s)$, $r[k] \doteq r(kT_s)$, where T_s is the sampling period.

The *adaptive law* recursively updates an estimate of H, $\hat{H}[k]$, based on the available measurements. The *high-level* *control law* takes the estimate $\hat{H}[k]$ and solves for $\hat{x}[k+1]$ such that

$$c\left(\hat{x}[k+1], \hat{H}[k+1]\hat{x}[k+1]\right) = r[k]. \tag{11}$$

It then sends $\hat{x}$ to a *low-level controller* which is responsible for obtaining

$$\lim_{t\to\infty} x = \lim_{k\to\infty} \hat{x}[k] \tag{12}$$

whenever the second limit exists.

The estimate of H is updated by the adaptive law according to the following convex optimization:

$$\begin{aligned} \hat{H}[k+1] = \arg\min_{H\in\mathcal{P}} \quad & \left\| H - \hat{H}[k] \right\|_F^2 \\ \text{subject to} \quad & \left\| y\left[k\right] - Hx\left[k\right] \right\|^2 \leq \delta \end{aligned}$$

where $\delta \in \mathbb{R}_{\geq 0}$ is a constant which depends on the measurement noise, and $\mathcal{P}$ is a set that must satisfy the following basic conditions:

a) The set $\mathcal{P}$ is convex.
b) The set $\mathcal{P}$ contains the true value of H.
c) For every $\hat{H} \in \mathcal{P}$ and $r \in \mathcal{R}$, there is a solution to (11).

The third condition can be relaxed, but that requires a more complex high-level control law than the one we use and present here. We proved in [3] that with these three components we can guarantee (5).

We apply these three components of the nonlinear adaptive control methodology to the network control problem we consider here as follows. We assume a two-way communication, all-to-one and one-to-all, from all the agents including the pseudo-agent to a central processing unit. The central processing unit implements the adaptive law and the high-level control law, and sends each $\hat{x}_i$, where $\hat{x}$ is computed numerically from $\hat{H}$, to the corresponding i'th agent (excluding the pseudo-agent). Each agent then sets $u_i(t) = \hat{x}_i[k]$ where k is such that $kT_s \leq t < (k+1)\,T_s$. By (2) this guarantees that $x \to \hat{x}$ if $\hat{x}$ converges (which is proved to hold by the results of [3]). Therefore the individual inner-loop controllers, collectively, fulfill the duty of the low-level controller. Note that this is true even though x_{n+1} is not controllable, since $\hat{x}_{n+1} = x_{n+1}$.

Remark 1: The results in [3] do not consider delays between the plant and the adaptive law and between the high-level controller and the low-level controller, delays that are present once communication is introduced. However, carefully analyzing these results show that they are insensitive to delays. The proof that $\hat{x}_k$ converges to some steady state value is independent of the behavior of the low-level controller, and therefore still holds in the presence of delays. Likewise, if (12) holds without delays, it will also hold if the low-level controller only receives a delayed version of $\hat{x}$. Therefore, delays will only results in a slower rate of convergence.

Regarding the dependency of y on $\dot{x}$ in (4) we note that it does not affect the steady-state solution, therefore we treat it as measurement noise which is handled by the theory of [3]. For the adaptive law we define $\mathcal{P}$ as

$$\mathcal{P} \doteq \left\{ \left[\begin{array}{cc} h_1 & \bar{h}_2 \\ h_2^T & 0 \end{array}\right] \in \mathbb{C}^{(n+1)^2} \;\middle|\; \begin{array}{l} h_1 \in \mathbb{C}^{n\times n} \quad h_2 \in \mathbb{C}^n \\ h_1 + \bar{h}_1^T \geq 0 \end{array} \right\} \tag{13}$$

where the inequality is to be interpreted in the semidefinite sense — the Hermitian matrix $h_1' + \bar{h}_1'^T$ is restricted to be positive semidefinite. We verify that the three basic conditions of $\mathcal{P}$ are satisfied. The semidefinite constraint is convex, therefore $\mathcal{P}$ is convex. Every impedance matrix z of a passive electric network is such that $z + \bar{z}^T$ is semidefinite positive [5]. While h_1 in (13) corresponds to $z' = \operatorname{diag}(\bar{\gamma})\, z \operatorname{diag}(\gamma)$ in (10), we still have that $z' + \bar{z}'^T$ is semidefinite positive if this property holds for z. Therefore the true value of H belongs to $\mathcal{P}$.

If $n = 1$, c and $r \in \mathcal{R}$ are defined as in (6)–(9), and

$$\hat{H} = \left[\begin{array}{cc} \hat{h}_1 & \bar{\hat{h}}_2 \\ \hat{h}_2 & 0 \end{array}\right],$$

then when

$$Q \doteq \left(\left|\hat{h}_2\right|^2 |x_{n+1}|^2 + 4p_1 \operatorname{Re}\hat{h}_1 \right) \Big/ \left(\left|\hat{h}_2\right|^2 |x_{n+1}|^2 \right)$$

is nonnegative a solution to $c(\hat{x}, \hat{H}\hat{x}) = r$ is given by

$$\hat{x}_1 = \frac{\bar{\hat{h}}_2 x_{n+1}\left(\sqrt{Q} - 1\right)}{2\operatorname{Re}\hat{h}_1} \qquad \hat{x}_{n+1} = x_{n+1}. \tag{14}$$

In our case, $p_1 \geq 0$ because power is to be generated, and $\hat{h}_1$, representing an electric network with passive elements, always has a nonnegative real part. Therefore Q cannot be negative and the existence of a solution is guaranteed. Note that (14) is continuous at $\operatorname{Re}\hat{h}_1 = 0$ where the solution is given by $\hat{x}_1 = \left(\bar{\hat{h}}_2 x_{n+1} p_1\right) \Big/ \left(\left|\hat{h}_2\right|^2 |x_{n+1}|^2\right)$. Supported by experimental results in which numerous scenarios were randomly generated, and for every one a numerical solution could be found, we postulate the following extrapolation for $n > 1$:

Conjecture 1: For any $H \in \mathcal{P}$ there is a solution to (11) if $p_i \geq 0$, $\forall i \in [n]$.

We note that an analytical solution for (11) does not exist for $n > 1$. If Conjecture 1 is true then $\mathcal{P}$ also satisfies the third condition.

IV. Limited Communication Approach

In order to assess the properties of our approach suggested above, we present another approach that employs more limited communication. It follows the idea of frequency regulation in automatic generation control [6, ch. 9], and uses standard doubly-fed induction generator (DFIG) control [1]. We assume a one-way, one-to-all communication, from the pseudo-agent to all other agents. From the measurements of the pseudo-agent the error $r_{n+1} - c_{n+1} = -\operatorname{Im} x_{n+1}\bar{y}_{n+1}$ in (7) is computed, and a proportional-integral (PI) controller is used to produce a new signal ρ as

$$\rho = -\frac{k_i' \operatorname{Im} x_{n+1}\bar{y}_{n+1}}{s} - k_p' \operatorname{Im} x_{n+1}\bar{y}_{n+1}$$

where k'_p and k'_i are the proportional and integral gains, respectively. The pseudo-agent then transmits ρ to all the agents through the communication channel. The logic behind this is that in the local region of interest around the steady-state solution, a region whose boundaries are yet to be precisely defined, an increase in $\operatorname{Im} y_i\bar{x}_i$, $i \in [n]$, results in an increase in $\operatorname{Im} x_{n+1}\bar{y}_{n+1}$. We then need that $y_i\bar{x}_i \to p_i + j\rho p_i$. To accomplish that we pass the conjugate of the error

$$e_i \doteq p_i + j\rho p_i - y_i\bar{x}_i, \qquad \forall i \in [n] \tag{15}$$

through a PI controller and rotate (in the complex plane) the result by the angle of y_i to produce u_i:

$$u_i = \frac{y_i}{|y_i|}\left(\frac{k_i\bar{e}_i}{s} + k_p\bar{e}_i\right). \tag{16}$$

The above unit control (16) is standard in control of DFIG.

When the y_i's (excluding y_{n+1}) are independent of the x_i's (excluding x_{n+1}), which happens when z becomes negligibly small corresponding to strong-grid conditions, the system-wide closed-loop control system becomes linear. With appropriate choice of the PI gains, a proof of stability can be easily derived. In weak-grid conditions, the system-wide control system becomes nonlinear, and no proof is currently available. Nevertheless, experimental results still show the success of this simple approach even in weak-grid conditions. As we show in the next section, however, the use of this approach with limited communication does negatively affect the performance.

Remark 2: An alternative control objective can be considered, in which the reactive power each turbine needs to produce is only a function of the active power at that turbine. For such an objective function, a fully distributed control approach using only unit controllers without communication can be implemented. However, the optimal amount of reactive power changes when other turbines generate more or less power, and when the network changes. Setting too much reactive power will put unnecessary strain on the generators, and setting too little may result in instability as the control objective will no longer be achievable.

V. Simulation Results

We evaluated the two approaches of §III and §IV on a network consisting of 5 agents. The dynamics of each agent is given by (3) with $a = 0.1$s. The impedance matrix of the network is:

$$z = \begin{bmatrix} 2z_0 & 2z_0 & \cdots & 2z_0 \\ 2z_0 & 3z_0 & \cdots & 3z_0 \\ \vdots & \vdots & \ddots & \vdots \\ 2z_0 & 3z_0 & \cdots & 6z_0 \end{bmatrix},$$

where $z_0 = 0.05 + 0.5j$. This corresponds to a network of wind turbines connected in series, where the first turbine is connected to the main grid with a transmission line of impedance $2z_0$, and then each turbine is connected to the previous one by a transmission line of impedance z_0. This high impedance value, typically not found in wind farm projects, was selected in order to better differentiate the two approaches. The matrix L is defined similarly to (10) with z_{ij} replaced by $\operatorname{Im} z_{ij}/(2\pi f)$ where $f = 60$Hz.

For the limited communication approach of §IV we added a transmission delay of 0.1 second between the pseudo-agent and the other agents. For the nonlinear adaptive control approach of §III we added a transmission delay of 0.1 second between the agents and the central processing unit, and another 0.1 second delay between the central processing unit and the agents. The results of both approaches are presented in Figure 1, where $p_i = 0.5$ $\forall i \in [n]$.

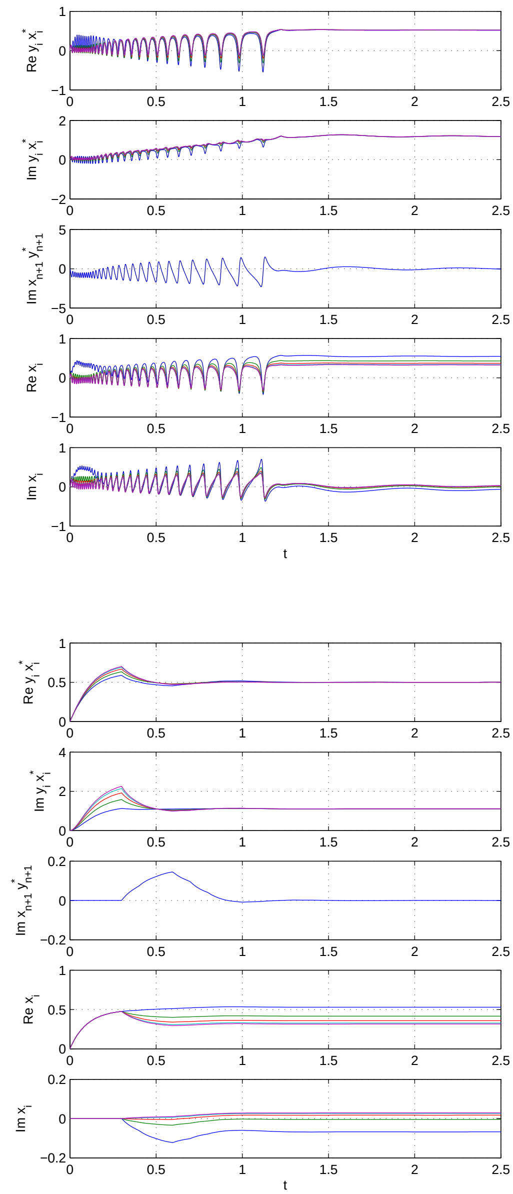

Fig. 1. Results comparing the approach of §IV in the top set of five plots, and the approach of §III in the bottom set. The plots, from top to bottom, are $\operatorname{Re} y_i\bar{x}_i$, $\operatorname{Im} y_i\bar{x}_i$, $\operatorname{Im} x_{n+1}\bar{y}_{n+1}$, $\operatorname{Re} x_i$, $\operatorname{Im} x_i$, $i \in [n]$. Each color represents a different agent.

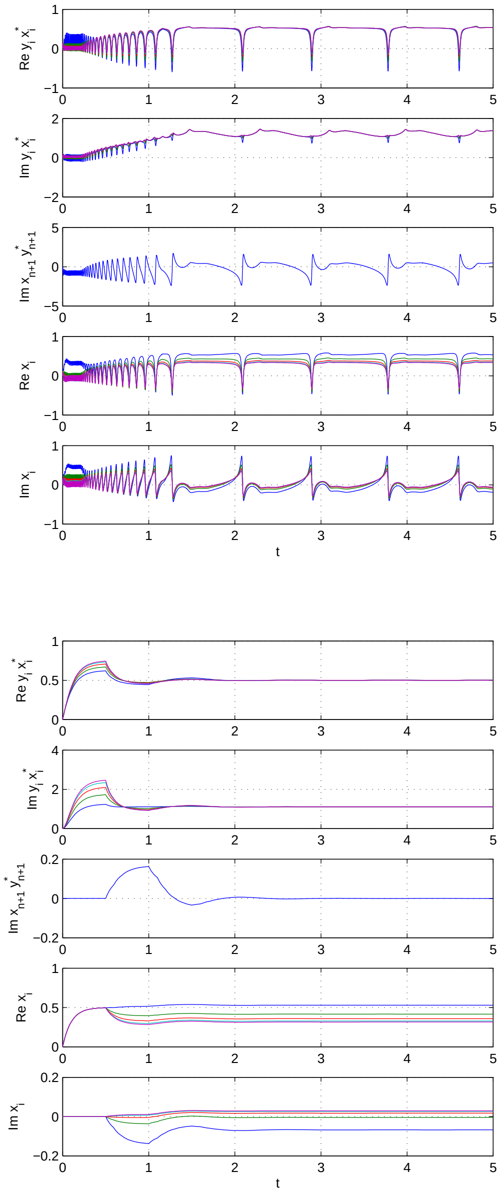

Fig. 2. Results comparing the approach of §IV in the top set of five plots, and the approach of §III in the bottom set, with doubled time delays.

The trajectory of the approach of §IV follows two distinct phases. It starts with a nonlinear phase until $t \approx 1.1$s, and then changes to a linear phase thereafter. We found the following selection of PI gains to minimize the duration of the nonlinear phase: $k_p = 8$, $k_i = 1$, $k_p' = 0$, $k_i' = 4$. A change in any of these individual gains leads to either instability, prolongation of the nonlinear phase, or more rapid oscillations during the nonlinear phase. While one can linearize the system around the steady-state solution and find other gains that result in faster convergence during the linear phase, our experience is that it only prolongs the nonlinear phase. Therefore existing linear tools for optimal gain selection might not be useful here.

Comparing the results of the two approaches, it is evident that the approach of §III converges faster and does not experience oscillations as in the nonlinear phase of the approach of §IV. Another important advantage of the approach of §III is robustness to time delays. We ran the same simulation but after doubling all the time delays. The results are shown in Figure 2. Note that the time scale in Figure 2 is twice longer than that of Figure 1. While with the approach of §III the consequence was only longer convergence time, as supported by its theory, with the approach of §IV the consequence was inability to converge at all.

VI. Conclusions

We considered a network control problem, where the control objective is to regulate an unknown nonlinear function of the state. We presented and evaluated two different approaches for addressing this problem. One is the extension to a network setting of a new nonlinear adaptive control scheme we developed specifically for regulating a nonlinear function of the state. Another, limited communication approach, is based on ideas from induction generator control and frequency regulation. While the limited communication approach is simpler to implement, requires less communication, and attains the control objective in certain conditions, the nonlinear adaptive control approach provides better performance over a wider range of conditions.

The main purpose of this paper is to present the application of the novel nonlinear adaptive control scheme for networked systems. Further theoretical study is required for the limited communication approach, providing proof of stability and means for optimal gain selections. Another purpose of this paper is therefore to foster further research into the limited communication approach.

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